

Laws of Exponents

Study Guide: Simplifying and Comparing Expressions

The problem

Is each expression equivalent to $((1/2)^3 \div 2^{-6}) \cdot 2^6$?

Big idea: get everything to the same base (here, base 2), then use the exponent rules.

1. The Rules You Need

Rule	In words	Example
Multiply, same base	Keep the base and ADD the exponents.	$2^3 \cdot 2^6 = 2^{3+6} = 2^9$
Divide, same base	Keep the base and SUBTRACT the exponents (top minus bottom).	$2^5 \div 2^2 = 2^{5-2} = 2^3$
Negative exponent	A negative exponent means “flip it”: take the reciprocal.	$2^{-3} = 1/2^3 = 1/8$
Fraction to a power	$1/2$ is the same as 2^{-1} , so $(1/2)^n = 2^{-n}$.	$(1/2)^3 = 2^{-3}$

2. Solving the Original Problem

Step	Work	Why
1	$(1/2)^3 = 2^{-3}$ so: $(2^{-3} \div 2^{-6}) \cdot 2^6$	Rewrite the fraction as a power of 2 so every term has the same base.
2	$2^{-3} \div 2^{-6} = 2^{-3 - (-6)} = 2^3$	Dividing means subtract exponents. Subtracting a negative is adding: $-3 + 6 = 3$.
3	$2^3 \cdot 2^6 = 2^{3+6} = 2^9$	Multiplying means add exponents. The base stays 2.
4	$2^9 = 512$	Evaluate: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 512$.

Answer: the original expression equals $2^9 = 512$

3. Quick Check With Real Numbers

Numbers never lie. Try the original problem with plain fractions and whole numbers:

$$(1/2)^3 = 1/8 \quad 2^{-6} = 1/64 \quad 2^6 = 64$$

$$(1/8) \div (1/64) = 8 \quad \text{then} \quad 8 \cdot 64 = 512 \quad \checkmark$$

It matches $2^9 = 512$, so the exponent rules worked.

4. Checking Each Expression

Target value to match: $2^9 = 512$

#	Expression	Equivalent ?	Justify
7	$((1/2)^3 \div (1/2)^6) \cdot 2^6$	YES	Since $2^{-6} = (1/2)^6$, this is the original written a different way. $(1/2)^{3-6} = (1/2)^{-3} = 2^3$, and $2^3 \cdot 2^6 = 2^9$.
8	2^{15}	NO	The exponents give $3 + 6 = 9$, not 15. Also, $2^{15} = 32,768$, which is far from 512.
9	$(2^3 \div 2^6) \cdot 2^6$	NO	$2^3 \div 2^6 = 2^{-3}$, then $2^{-3} \cdot 2^6 = 2^3 = 8$. The top is 2^3 , but the original has $(1/2)^3$. The fraction got flipped, and $8 \neq 512$.
10	$(2^{-3} \div 2^{-6}) \cdot 2^6$	YES	$2^{-3} \div 2^{-6} = 2^3$, and $2^3 \cdot 2^6 = 2^9$. This is the original with $(1/2)^3$ rewritten as 2^{-3} .

5. Watch Out For These

- Subtracting a negative. $-3 - (-6)$ is the same as $-3 + 6$. Rewrite it with a plus sign before you calculate.
- Keep the base the same. When you multiply or divide powers, only the exponents change. 2^6 must not turn into 4^6 (that equals 2^{12} and changes the answer).
- Fractions flip the sign. $(1/2)^3 = 2^{-3}$, not 2^3 . A half is 2 to the NEGATIVE one.
- Say "add" or "subtract" out loud before you do the exponent work. Multiply means add. Divide means subtract.

6. Try It Yourself

Use the same steps: same base, then add or subtract exponents.

1	$5^{-2} \cdot 5^6$	Answer:
2	$3^4 \div 3^{-2}$	Answer:
3	$((1/3)^2 \div 3^{-4}) \cdot 3^2$	Answer:

Answers (cover until you've tried them): 1) $5^4 = 625$ 2) $3^6 = 729$ 3) $3^4 = 81$